Acceleration / Generation of Large-scale Flows in Astrophysical Objects

Nana L. Shatashvili[1],

In collaboration with S. M. Mahajan[2] & Z. Yoshida[3]

(1) Department of Physics, Faculty of Exact & Natural Sciences *and*
TSU Andronikashvili Institute of Physics,
Javakhishvili Tbilisi State University, Georgia
(2) Institute for Fusion Studies, The University of Texas, Austin, USA
(3) Graduate School of Frontier Sciences, The University of Tokyo, Japan

Based On:

1. S. Ohsaki, N.L.Shatashvili, Z. Yoshida and S.M. Mahajan. *The Astrophys. J*. **559**, L61 (2001); **570**, 395 (2002)
2. S.M. Mahajan, N.L. Shatashvili, S.V. Mikeladze & K.I. Sigua. *The Astrophys. J.* **634**, 419 (2005)
3. S.M. Mahajan, K.I. Nikol'skaya, N.L. Shatashvili & Z. Yoshida. *The Astrophys. J.* **576**, L161 (2002)
4. S.M. Mahajan, N.L. Shatashvili, S.V. Mikeladze & K.I. Sigua. *Phys. Plasmas*. **13**, 062902 (2006)
5. Z. Yoshida & N.L. Shatashvili. *AIP Conf. Proc*. **1392**, 73 (2011); *ArXiv:*1105.5281v1 [astro-ph.GA] (2011)
6. Z. Yoshida & N.L. Shatashvili. *ArXiv:*1210.3558v1 [physics.flu-dyn] (2012)

Outline

- **Acceleration of *particles* – *sources of free energy for acceleration***
- **Acceleration / Generation of *Large Scale Flows* - *sources of energy for acceleration***
- **Magneto-Fluid coupling – model equations – *Quasi-equilibrium approach***
- **Acceleration / Generation of flows - incompressible plasma case –**
  Catastrophe
- **Acceleration / Generation of flows - incompressible plasma case –**
  Reverse Dynamo
- **Acceleration / Generation of flows - compressible plasma case**
- **Acceleration / Generation of flows - compressible rotational neutral fluid case**
  – *Generelized Beltrami Flow – a model of disk-jet system*
- **Summary & Conclusions, *perspectives***

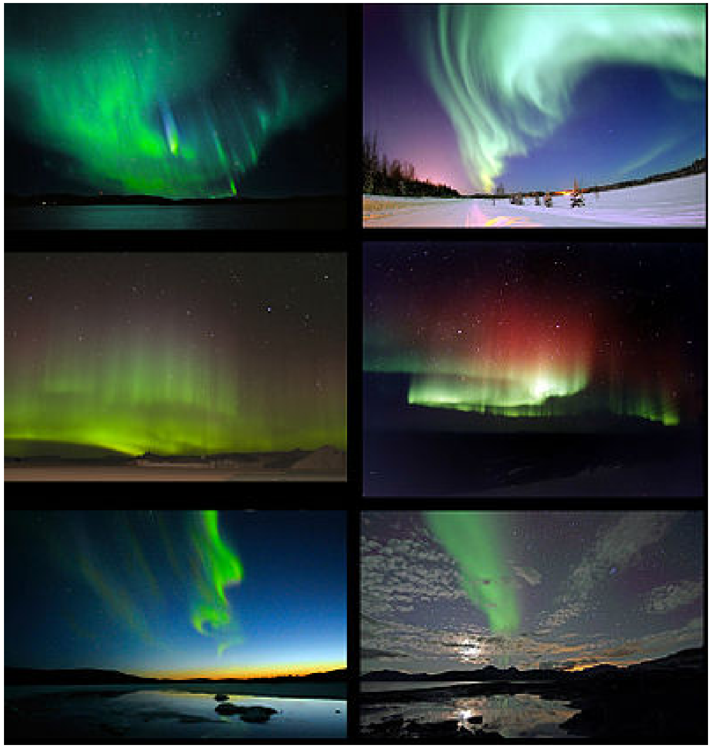
Acceleration of *particles*

- **High-energy particle acceleration is observed in a diverse variety of laboratory devices as well as astrophysical sites ranging from the *terrestrial aurorae* to the most distant *quasars*.**

- **In astrophysical sites particle acceleration is a fairly common channel for the release of large-scale kinetic, rotational and magnetic energy.**

- **Physical mechanisms include electrostatic acceleration, stochastic processes and diffusive shock energization.**

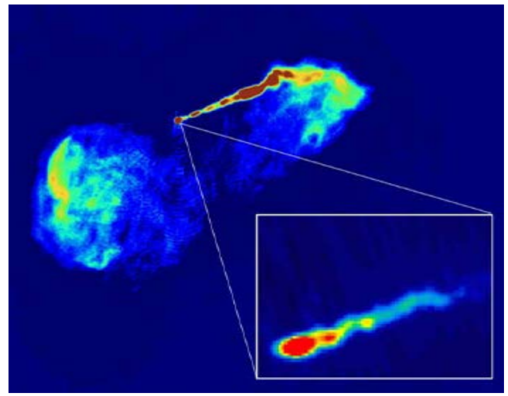
- **The overall acceleration efficiency is controlled by the low energy particle injection.**

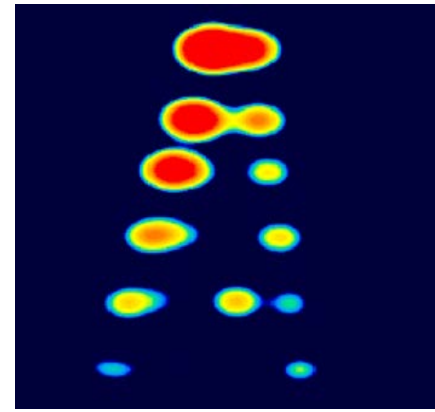


Aurora Australis & Aurora Borealis **from around the world, *including those with rarer blue and red lights***

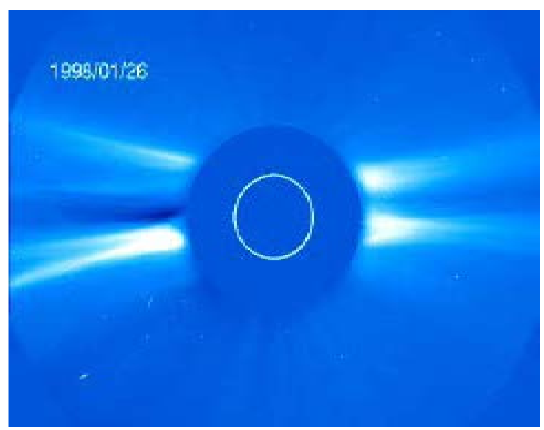
Acceleration / Generation of *Large Scale Flows*

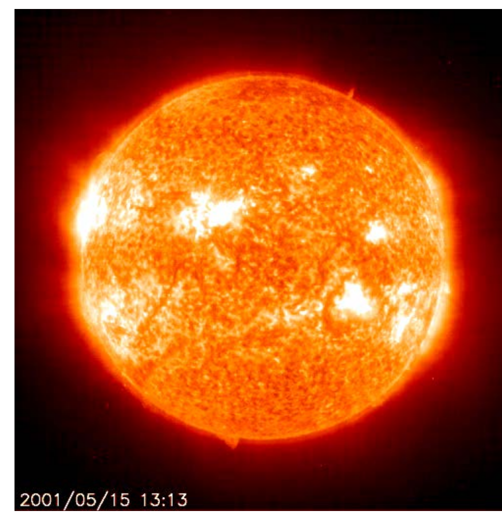
Acceleration of large-scale flows - creation of stellar winds, variety of outflows, jets, & etc. – often observed **in astrophysical objects.**





Recent observations - **solar corona is a highly dynamic arena replete with multi-species multiple–scale spatiotemporal structures.**





Acceleration / Generation of *Large Scale Flows - 2*

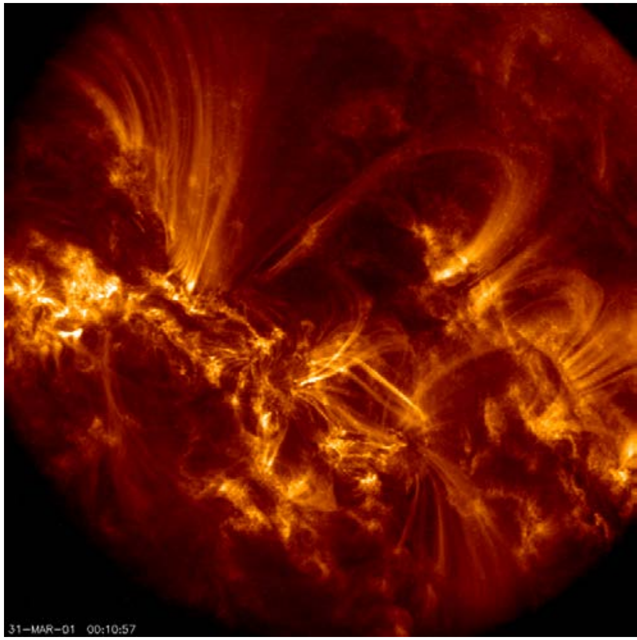
Strong flows are found everywhere in the low Solar atmosphere — in the sub-coronal (*chromosphere*) as well as in coronal regions – *recent observations from HINODE (De Pontieu et al. 2011,20122).*

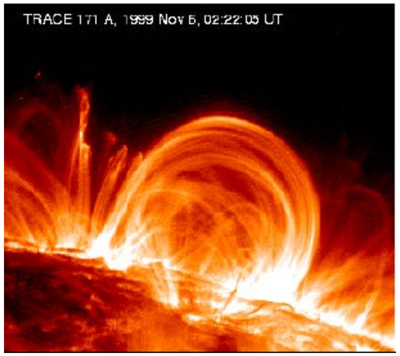
Co-existing dynamic structures: Flares; Spicules; Different-scale dynamic closed/open structures .

Message: Different temperatures, Different life-times

Indication: Any particular mechanism may be dominant in a specific region of parameter space.

Equally important: ***the plasma flows may complement the abilities of the magnetic field in the creation of the amazing richness observed in the Atmosphere***





Sources of energy for *large-scale plasma flow* acceleration

The most obvious process for acceleration (***rotation is ignored***):

the **conversion of**

- **magnetic**
- **and/or the thermal energy**
- **turbulence energy**

=== > **to plasma kinetic energy**

- **Magnetically driven transient but sudden flow–generation models:**
- **Catastrophic models**
- **Magnetic reconnection models**
- **Models based on instabilities**

Quiescent pathway:

- ***Bernoulli mechanism converting thermal energy into kinetic***
- **General magneto-fluid rearrangement of a relatively constant kinetic energy:** going from an initial *high density–low velocity* to *a low density–high velocity state.*

Magneto-Fluid coupling – model equations

Minimal two-fluid model – **incompressible, constant density Hall MHD** – ***gravity & rotation are ignored.***

Dimensionless system in standard Alfvenic units.

Velocities - normalized to the Alfven speed with appropriate magnetic field.

Times - measured in terms of the (cyclotron time)$^{-1}$, **Lengths** - to collisionless skin depth λ_{i0}.

Defining equations are:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left[[\boldsymbol{V} - \boldsymbol{\nabla} \times \boldsymbol{B}] \times \boldsymbol{B} \right], \qquad \boldsymbol{V}_e = \boldsymbol{V} - \boldsymbol{\nabla} \times \boldsymbol{B} \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial t} = \boldsymbol{V} \times (\boldsymbol{\nabla} \times \boldsymbol{V}) + (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} \left(P + \frac{V^2}{2} \right) \tag{2}$$

- **The red terms are due to Hall current** and the **blue terms are vorticity forces**.

Quasi-equilibrium Approach

Assumption (gravity included): there exists fully ionized & magnetized plasma structures ==> the quasi–equilibrium two–fluid model will capture the essential physics of flow acceleration

Simplest two–fluid equilibria: $T = \mathrm{const} \longrightarrow \mathrm{n}^{-1}\nabla \mathrm{p} \rightarrow \mathrm{T}\nabla \ln \mathrm{n}.$ **Generalization to homentropic fluid is straightforward.**

The dimensionless equations for compressible case: *Mahajan et al. 2001, PoP*

$$\frac{1}{n}\nabla \times \mathbf{b} \times \mathbf{b} + \nabla\left(\frac{r_{A0}}{r} - \beta_0 \ln\, n - \frac{V^2}{2}\right) + \mathbf{V} \times (\nabla \times \mathbf{V}) = 0, \tag{3}$$

$$\nabla \times \left[\left(\mathbf{V} - \frac{\alpha_0}{n}\nabla \times \mathbf{b}\right) \times \mathbf{b}\right] = 0 \tag{4}$$

$$\nabla \cdot (n\mathbf{V}) = 0 \quad (5) \qquad \nabla \cdot \mathbf{b} = 0 \quad (6)$$

Parameters: $r_{A0} = GM/V_{A0}^2 R_0 = 2\beta_0\, r_{c0}, \quad \alpha_0 = \lambda_{i0}/R_0, \quad \beta_0 = c_{s0}^2/V_{A0}^2,$

$\lambda_{i0} = c/\omega_{i0}$ - the collisionless ion-skin depth, **are defined by** $n_0,\ T_0,\ B_0$

Hall current contributions are significant when $\alpha_0 > \eta$, (η - inverse Lundquist number)

Important in: interstellar medium, early universe, white dwarfs, neutron stars, stellar atmosphere.
Typical solar plasma: condition is easily satisfied.

Hall currents modifying the dynamics of the microscopic flows/fields - have a profound impact on the generation of macroscopic magnetic fields & macroscopic flows

The double Beltrami solutions are

$$\mathbf{b} + \alpha_0 \nabla \times \mathbf{V} = d\ n\ \mathbf{V}, \qquad \mathbf{b} = a\ n\ \left[\mathbf{V} - \frac{\alpha_0}{n} \nabla \times \mathbf{b}\right], \tag{7}$$

a and ***d*** — **dimensionless constants related to ideal invariants:**

the Magnetic helicity

$$h_1 = \int (\mathbf{A} \cdot \mathbf{b})\ d^3x, \tag{8}$$

& the Generalized helicity

$$h_2 = \int (\mathbf{A} + \mathbf{V}) \cdot (\mathbf{b} + \nabla \times \mathbf{V}) d^3x. \tag{9}$$

obeying the ***Bernoulli Condition***

$$\nabla \left(\frac{2\beta_0 r_{c0}}{r} - \beta_0 \ln n - \frac{V^2}{2} \right) = 0, \tag{10}$$

relating the density with the flow kinetic energy & gravity.

Quasi-equilibrium → Eruptive and Explosive events, Flaring

Incompressible case

The parameters of the DB field change ***– assumption***

- **the parameter change is sufficiently slow / adiabatic.**
- **at each stage, the system can find its local DB equilibrium.**
- **in slow evolution the dynamical invariants: h_1, h_2, & the total (magnetic + fluid) energy E are conserved.**

The General equilibrium solution ***for incompressible*** **case is shown to be**

(***G_λ , G_μ - solutions of Beltrami equation*****)**

$$\boldsymbol{b} = C_\mu \boldsymbol{G}_\mu(\mu) + C_\lambda \boldsymbol{G}_\lambda(\lambda), \tag{11}$$

$$\boldsymbol{V} = \left(\frac{1}{a} + \mu\right) C_\mu \boldsymbol{G}_\mu(\mu) + C_\lambda \left(\frac{1}{a} + \lambda\right) \boldsymbol{G}_\lambda(\lambda). \tag{12}$$

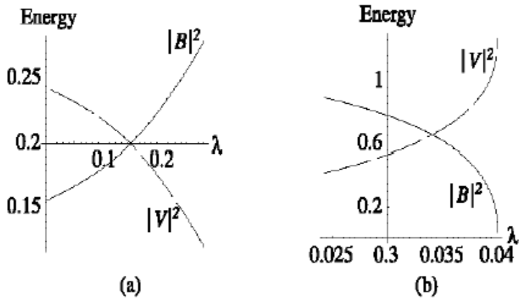
The catastrophic loss of equilibrium may occur in one of the following two ways:

1. The ***determining length scales*** (λ - large-scale, μ - short-scale) ***for the field variation***, **go from being real to complex**.

2. **amplitude of either of the 2 states (** ***G_λ , G_μ*** **) ceases to be real**.

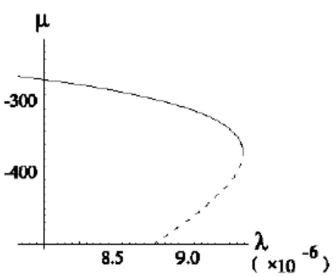
- **Large scale λ – control parameter — observationally motivated choice.**
- ***Example*: structure–structure interactions (2D Beltrami ABC field with periodic boundary conditions). Choosing real λ, μ for quasi–equilibrium.**
- **Conditions for catastrophic changes in Slowly Evolving Solar Structures (sequence of DB states) leading to a fundamental transformation of the initial state are derived as:**
- **For** $E > E_c = 2\left(h_1 \pm \sqrt{h_1 h_2}\ \right)$ **the DB equilibrium suddenly relaxes to a SB state corresponding to the large macroscopic size.**
- **All of the short–scale magnetic energy is catastrophically transformed to the flow kinetic energy. *Seeds of destruction lie in the conditions of birth.***
- **The proposed mechanism for the energy transformation work in all regions of Solar Atmosphere with different dynamical evolution depending on the *Initial & Boundary Conditions* for a given region.**

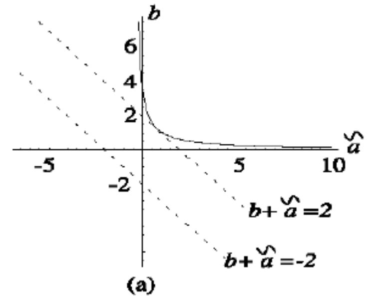
Large Scale Plasma flow acceleration – catastrophe ($n = const$)

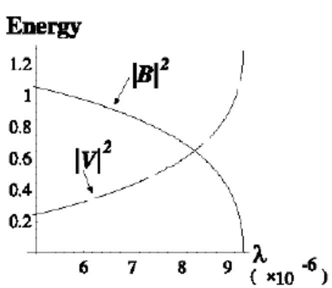


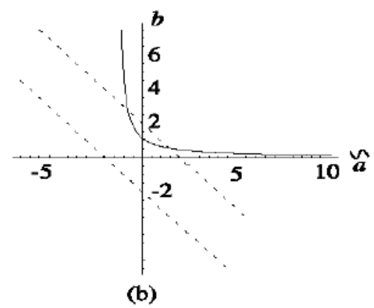
a) No catastrophe initial conditions

b) Catastrophe initial conditions





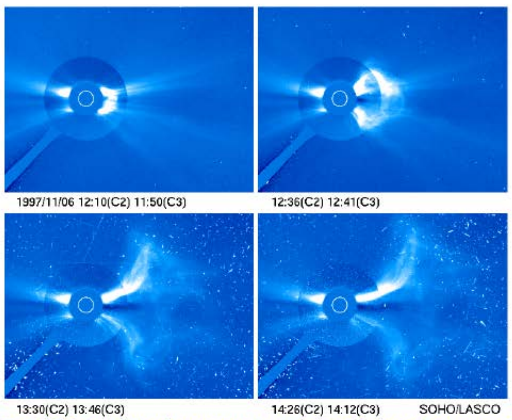




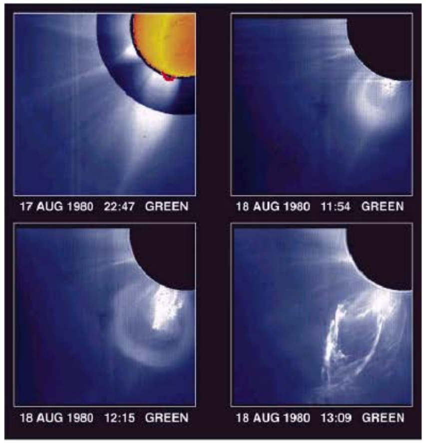
Solar Atmosphere:
Almost all initial magnetic energy (short scale) is transferred to flow

Root coalescence:
No separation between roots at the transition!

Coronal Mass Ejection



SOHO/LASCO images of a coronal mass ejection on 6 November 1997



A time sequence of *Solar Maximum Mission* coronagraph images showing a CME on August 18, 1980 (from Hundhausen (1999))

Quasi-equilibrium → Eruptive and Explosive events, Flaring

Compressible case

Closed HMHD system (3-6) of equilibrium equations ($g(r) = r_{c0} / r$) ===>

1D simulation - a variety of boundary conditions: Flow with 3.3 km/s ends up with ~ 100 km/s .

For small α_0 there exists some height where density drops sharply with a corresponding sharp rise in the flow speed ===>

- **There is a catastrophe in the system.**
- **The distance over which it appears is determined by the strength of gravity $g(z)$.**
- **Amplification of flow is determined by local β_0 .**

If density fall is at a much slower rate than the slow scale 1D problem solution gives:

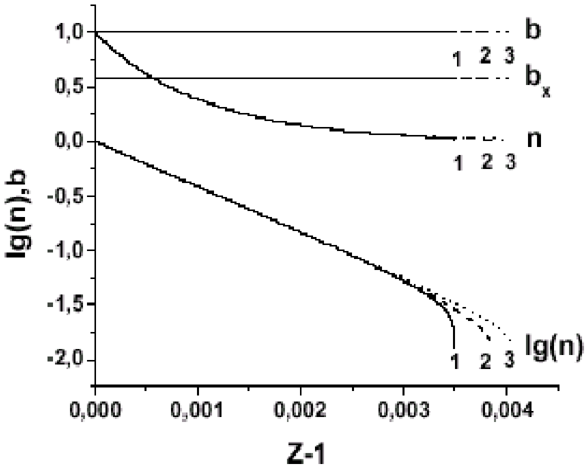
(if ***a ~ d >>1***; or ***a ~ d << 1*** , or *an equation of state is assumed, **T≠const***):

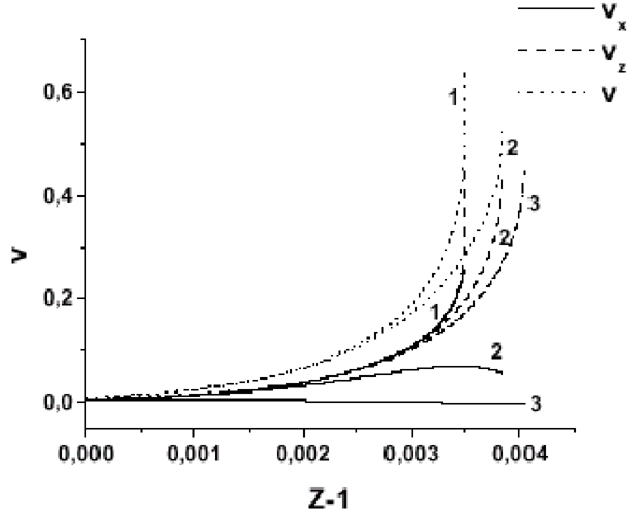
$$|V_{max}| = \frac{1}{d\, n_{min}} \qquad |V|^2 = 1/d^2n^2 \qquad |\mathbf{b}|^2 = const \qquad (13)$$

The Bernoulli condition transforms to the defining differential equation for density ==>

$$n_{min} = (2\beta_0)^{-1/2} d^{-1} \qquad (14)$$

Plasma flow acceleration – catastrophe ($n \neq const$)





Sub-Alfvénic flows: Boundary conditions at:

$Z_0 > (1+2.8\bullet10^{-3})R_s$ – the influence of ionization can be neglected

$|b_0| = 1$, $V_0 = 0.01V_{A0}$ (with $V_{x0} = V_{y0} = V_{z0}$)

DB parameters:
$d \sim a \sim 100$, $(a-d)/a^2 \sim 10^{-6}$

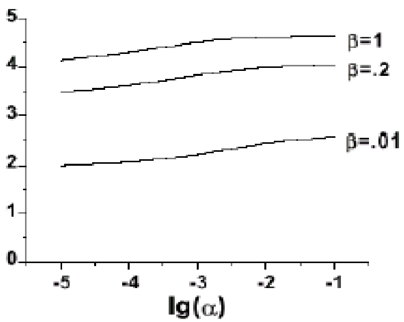
3 sets of curves labeled by α_0 for parameters versus height (Z- 1).
1- 2- 3 correspond to: $\alpha_0 = 0.000013; 0.005; 0.1$

(a) Blowup distance

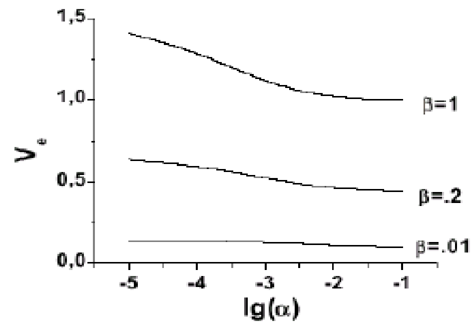
(b) velocity

vs.. α_0

(a)



(b)



Following are the (n_0 ; B_0 ; T_0 ; V_{A0}):

$10^{11}cm^{-3}$; 100G; 5eV; 600km/s; $\beta_0 \sim 0.007 << 1$

$|b|^2 \sim const$; Density fall → Velocity increase

Catastrophe!

Acceleration is determined by local β_0

Steady flow generation /acceleration – *Reverse Dynamo*

The Dynamo mechanism - generic process of generating macroscopic magnetic fields from an initially turbulent system - *biggest industry in plasma astrophysics + fusion.*

Standard Dynamo – generation of macro-fields from (primarily microscopic) velocity (*Flow Dominated Dynamo - FDD*) & magnetic (*Magnetically Dominated Dynamo - MDD*) fields.

Latest understanding - coupling of FDD & MDD at different heights (going from lower scale structures to larger scale structures).

Kinematic dynamo – the velocity field is externally specified & is not a dynamical variable!

”Higher” theories – MHD, Hall MHD, two fluid etc - the velocity field evolves just as the magnetic field does – the fields are in mutual interaction.

A question – A possible inference:

If short–scale turbulence can generate large–scale magnetic fields,
then short-scale turbulence should also be able to generate large-scale velocity fields.

Process of conversion of short–scale kinetic energy to large–scale magnetic → ”Dynamo” (D)

The mirror image process - conversion of short–scale magnetic energy to large–scale kinetic energy → ”Reverse Dynamo” (RD)

Extending the definitions:

- **Dynamo (D) process -** Generation of large-scale magnetic field from **any mix** of short–scale energy (magnetic & kinetic).
- **Reverse Dynamo (RD) process -** Generation of large-scale flow from **any mix** of short–scale energy (magnetic & kinetic).

Theory and simulation show

(1) D & RD processes operate simultaneously
(2) The composition of the turbulent energy determines the ratio of the large-scale flow / large-scale magnetic field

Micro (*short-scale*) and Macro (*large-scale*) Fields

The total fields in Eq.-s (1), (2) are broken into ambient & generated.

The **generated fields -** further split into **macro** & **micro** fields**:**

$$\mathbf{B} = \mathbf{b}_0 + \mathbf{H} + \mathbf{b}$$
$$\mathbf{V} = \mathbf{v}_0 + \mathbf{U} + \mathbf{v}$$

$\mathbf{b}_0$, $\mathbf{v}_0$ **- equilibrium,** $\mathbf{H}$, $\mathbf{U}$ **- macroscopic,** $\mathbf{b}$, $\mathbf{v}$ **- microscopic fields.**

Traditional dynamo theories - the short scale velocity field $\mathbf{v}_0$ is dominant.

We shall not introduce any initial hierarchy between $\mathbf{v}_0$ & $\mathbf{b}_0$.

We shall develop the natural unified Flow–Field theory.

Equilibrium – Initial State

Departure from the standard dynamo approach - our choice of the initial plasma state.

$$\nabla(p_0 + \boldsymbol{v}_0{}^2/2) = const$$

Equilibrium fields - the DB pair obeying Bernoulli condition

which may be solved in terms of the Single Beltrami (SB) states given by (11) & (12) for equilibrium fields.

Below: λ **- micro-scale,** μ **- macro-scale;** $|b| \ll b_0, \quad |v| < v_0$

Primary interest – to create macro fields from the ambient microfields.

Constructing the closure model of the Hall MHD eq-s & assuming that the original equilibrium is predominantly short-scale (from the DB fields we keep only the λ - part)

$$\boldsymbol{v}_0 = \boldsymbol{b}_0\left(\lambda + a^{-1}\right) \qquad \boldsymbol{v}_{e0} = \boldsymbol{v}_0 - \nabla\times\boldsymbol{b}_0 = \boldsymbol{b}_0\, a^{-1} \tag{15}$$

Straightforward algebra for isotropic ABC initial flow ===>

H **evolves independently of** ***U*** **but evolution of** ***U*** **does require knowledge of** ***H*****.**

Working out the nonlinear solution in linear clothing (*neglecting NLN terms*) we find:

$$U = \frac{q}{(s+r)} H \tag{16}$$

q, s, r **– fully defined by DB parameters & initial turbulent energy** b_0^2 .

A few remarkable features of linear solution:

- A choice of ***a; d*** (& hence of λ) **fixes relative amounts of microscopic energy in ambient fields**

  ==> also **fixes the relative amount of energy in the generated macroscopic fields *U* & *H***.

- **The linear solution makes NLN terms strictly zero – it is an exact (a special class) solution of the NLN system ==> remains valid even as *U* & *H* grow to larger amplitudes**

  (*appears in Alfvénic systems: MHD - nonlinear Alfvén wave:* Walen 1944,1945; in HMHD - Mahajan & Krishan, 2005)

(i) Analytical Results — *An Almost Straight Dynamo*

$a \sim d \gg 1$, **inverse micro scale micro-scales fields are** $\lambda \sim a \gg 1$ ===>

$v_0 \sim a\, b_0 \gg b_0$ **the ambient micro-scales fields are primarily kinetic.**

Generated macro-fields have opposite ordering - $U \sim a^{-1} H \ll H$

super-Alfvénic "turbulent flows" lead to steady flows (equally sub–Alfvénic)

(ii) Analytical Results — *An Almost straight Reverse Dynamo*

$a \sim d \ll 1$ $\lambda \sim a - a^{-1} \gg 1$ $v_0 \sim a\, b_0 \ll b_0$

The ambient energy is mostly magnetic; from a strongly sub-Alfvénic turbulent flow

the system generates a strongly super-Alfvénic macro–scale flow

$$U \sim a^{-1} H \gg H$$

D, RD Summary:

- **Dynamo and ”Reverse Dynamo” mechanisms have the same origin – are manifestation of the magneto-fluid coupling**

- ***U*** and ***H*** are generated simultaneously and proportionately. **Greater the macro-scale magnetic field (generated locally), greater the macro-scale velocity field (generated locally)**

- **Growth rate of macro-fields is defined by DB parameters (by the ambient magnetic & generalized helicities) and scales directly with ambient turbulent energy** $\sim b_0^2 \ (v_0^2)$.

- **The composition of the ambient turbulent energy determines the ratio of the large-scale flow / large-scale magnetic field.**

- **Impacts:** on the evolution of large-scale magnetic fields and their opening up with respect to fast particle escape from stellar coronae; on the dynamical and continuous kinetic energy supply of plasma flows observed in astrophysical systems.

A simulation Example for Dynamical Acceleration

Caution: Initial and final states have finite helicities (magnetic and kinetic).

The helicity densities are dynamical parameters that evolve self-consistently during the flow acceleration.

Rotation, dissipation & heat flux as well as compressibility effects were neglected!

2.5D numerical simulation of the general two-fluid equations in Cartesian Geometry.
Code: Mahajan et al. PoP 2001, Mahajan et al, 2005

Simulation system contains:

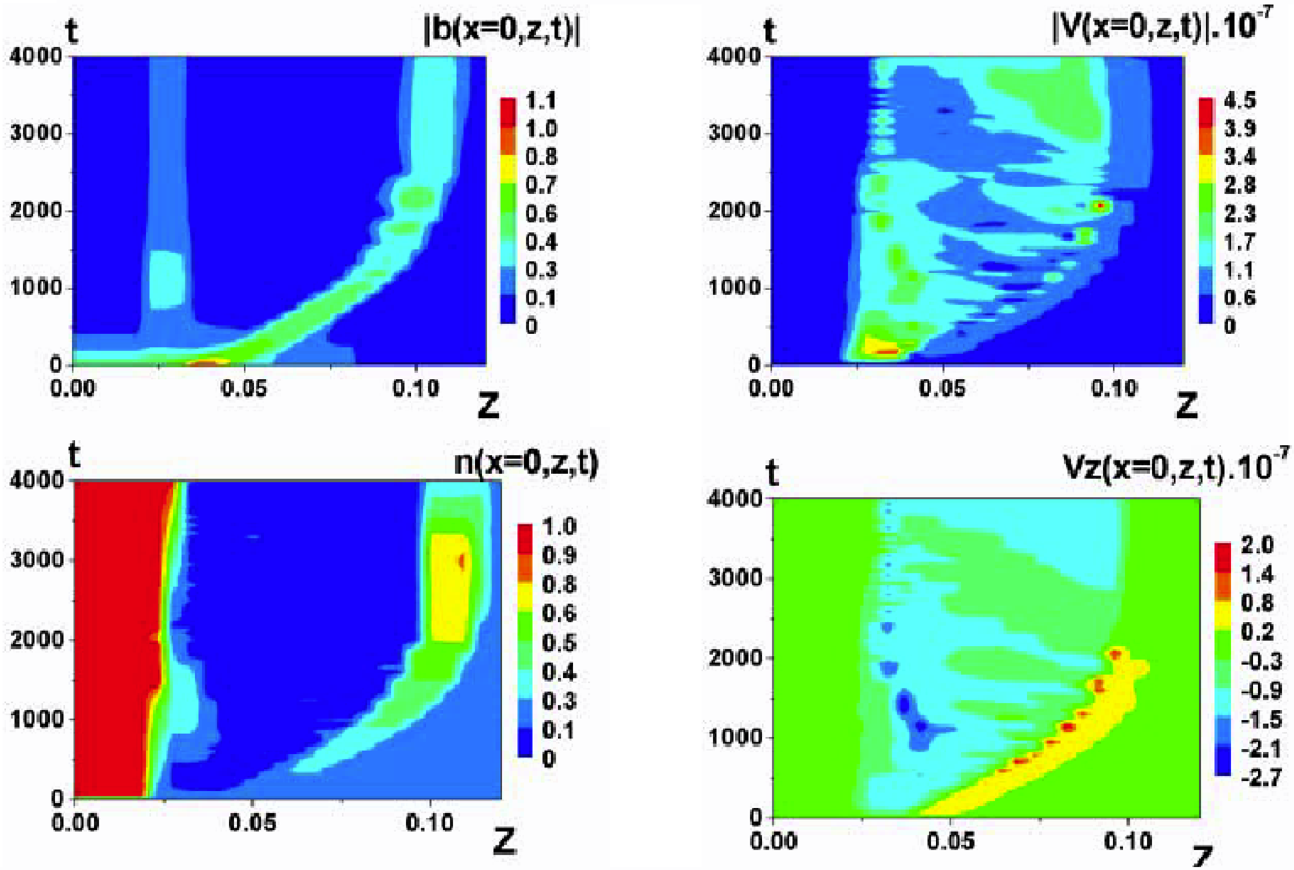
- **an ambient macroscopic field**
- **effects not included in the analysis**:
  1. dissipation and heat flux
  2. plasma is compressible embedded in a gravitational field $\rightarrow$ **extra possibility for micro-scale structure creation**.

Transport coefficients are taken from Braginskii and are local.

Diffusion time of magnetic field > duration of interaction process (would require T ≤· a few eV -s).

Study of trapping and amplification of a weak flow impinging on a single closed-line magnetic structure ===>

Dynamo and Reverce Dynamo Phenomena
In the center of the original closed magnetic field structure



Dynamical Emergence of the new magnetic field in region different from original; flux moves to the upper heights with time!

Accelerated flow follows the maximum field localization area – RD! **D & RD phenomena have oscillating/pulsating character.**

Generated field maximum ~0.5b$_0$; accelerated flow max. radial speed ~200km/s ; at ~2000sec time flow converts to down-flow!

Simulation Summary:

- **Dissipation present: Hall term *(through the mediation of micro-scale physics)* plays a crucial role in acceleration / heating processes.**

- **Initial fast acceleration in the region of maximum original magnetic field + the creation of new areas of macro-scale magnetic field localization with simultaneous transfer of the micro-scale magnetic energy to flow kinetic energy = manifestations of the combined effects of the D and RD phenomena**

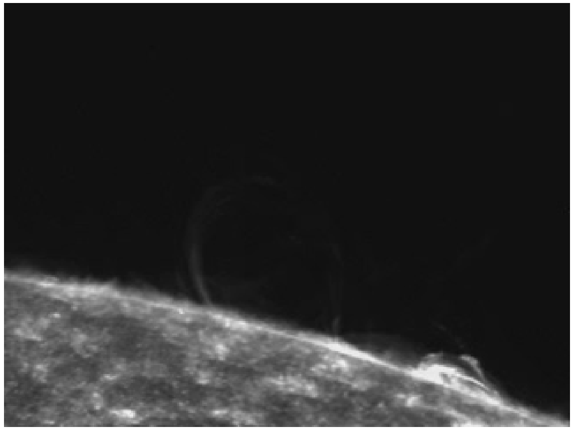
- **Continuous energy supply from fluctuations (dissipative, Hall, vorticity) ===> maintenance of quasi-steady flows for significant period**

- **Simulation: actual h_1, h_2 are dynamical.**

  Even if they are not in the required range initially, their evolution could bring them in the range where they could satisfy conditions needed to efficiently generate flows ===>

  several phases of acceleration

- **In the presence of dissipation, these up-flows play a fundamental role in the heating of the finely structured stellar atmospheres; their relevance to the solar wind is also obvious.**



Formation / primary heating of Solar coronal loopby up-flows; flow remains along loop, just slowed down – Mahajan at al (2001)

Generalized Beltrami flow – a model of disk-jet system (*rotation included*)

The macroscopic disk-jet geometry - a marked similarity

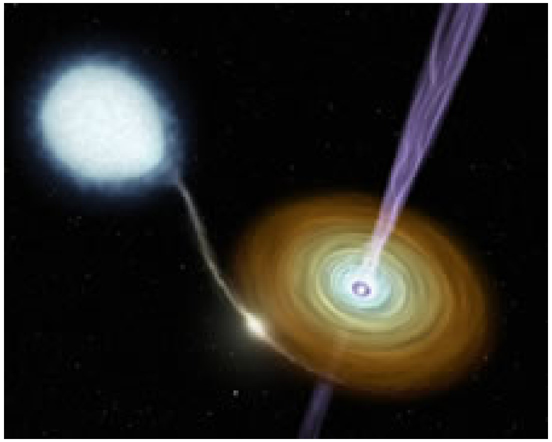
An accretion disk (AD) often combines with spindle-like jet of ejecting gas

&

constitutes a typical structure that accompanies a massive object of various scales, ranging from young stars to AGN.

The mechanism that rules each part of different systems - not universal.

Elliptical Galaxy M87 Emiiting Relativistic Jet



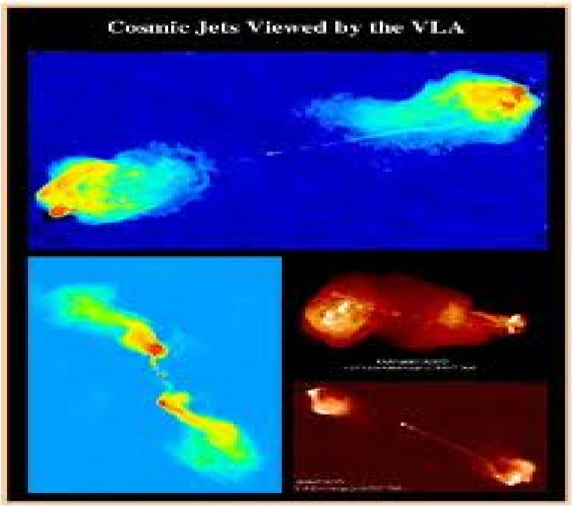
Neutron –Star Jet

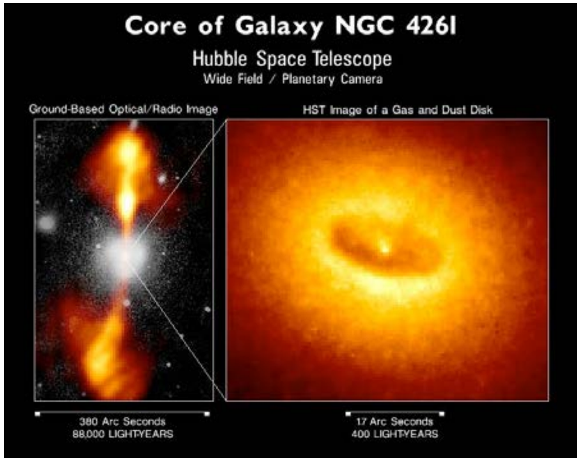
The macroscopic disk-jet geometry - a marked similarity - 2

Since early 70s (the discovery of radio galaxies & quasars) the main evidence from detecting **jets in different classes of astrophysical systems** observed to produce collimated jets near the massive central object - **the direct association with an AD** ***(reflecting different accretion regimes)***.

The opposite is not true in some objects for which AD-s do not require collimated jets ***(viscous transport/disk winds play the similar role in the energy balance).***

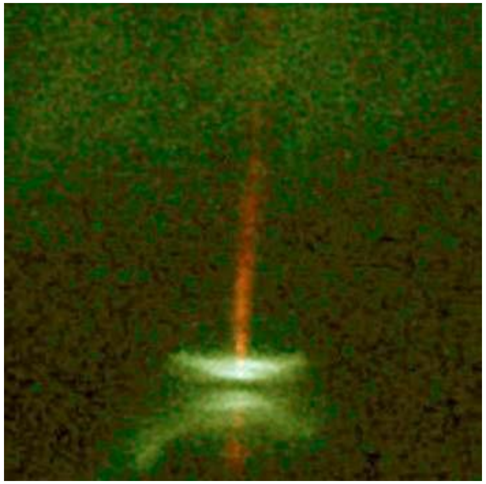
The macroscopic disk-jet geometry - a marked similarity despite the huge variety of the scaling parameters (*Lorentz factor, Reynolds number, Lundquist number, ionization fractions, etc.)*





Jets in AGN

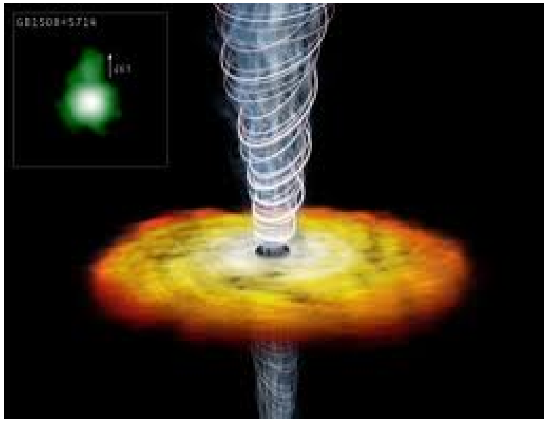
The basic properties of disk-jet systems



In the disk region:

- **transport processes of mass, momentum & energy depend strongly on the scaling parameters.**
- the **classical *(collisional)* processes are evidently insufficient to account for the accretion rate**, thus **turbulent transports *(involving magnetic perturbations)* must be invoked**.
- **winds may also remove the angular momentum from Disk.**

The connection of the disk & the jet is more complicated:



- **mass & energy of jet are fed by the accreting flow**, the mechanism & process of transfer are still not clear.
- **the major constituent of jets is the material of an AD surrounding the central object**.

For the fastest outflows the contributions to the total mass flux may come from outer regions as well. In AGN one may think of taking some energy from the central black hole.

Role of Magnetic Field

Magnetic fields are considered to play an important role in defining the local accretion.

- **When magnetic field is advected inwards by accreting material or/and generated locally** by some mechanism, the centrifugal force due to rotation may boost **jet along the magnetic field lines** up to a super-Alfvénic speed.

- **AGN - there is an alternative idea** suggested by *Blandford & Znajek (1977)* **based on electro-dynamical processes extracting energy from a rotating black hole.**

- **Extra-galactic radio jets might be accelerated by highly disorganized magnetic fields.**

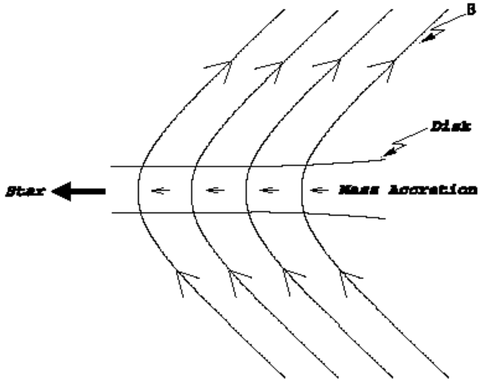
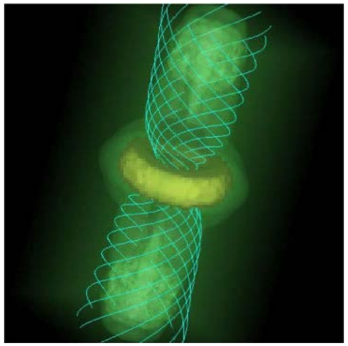
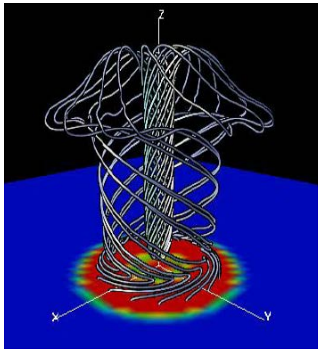


Fig. 1.—Schematic diagram of accretion flow in a disk threaded by magnetic flux accumulated by the process of star formation.





In addition to the energetics to account for the acceleration of ejecting flow, we have to explain how the streamlines/magnetic-field lines change the topology through the disk-jet connection.

Momentum Equation – Simplest MHD Model

Despite the diversity & complexity of holistic processes, there must yet be a simple and universal principle that determines the geometric similarity of disk-jet compositions!

Let's invoke a simple model for neutral fluid ($\boldsymbol{P} = \rho \boldsymbol{V}$ momentum density);
Momentum equation reads as:

$$\partial_t \boldsymbol{P} + \nabla \cdot (\boldsymbol{V}\,\boldsymbol{P}) = -\rho \nabla \phi - \nabla p - \nabla \cdot \mathbf{\Pi}, \qquad (17)$$

$\boldsymbol{\rho}$ - the mass density, $\boldsymbol{V}$ - flow velocity,
$\boldsymbol{\Pi}$ - the effective viscosity tensor, $\boldsymbol{\phi}$ - the gravity potential.

Energy densities are normalized by unit kinetic energy $\mathcal{E}_0 = \frac{\rho_0 V_0^2}{2}.$

Scale parameter is $\epsilon = \frac{\delta_i}{L_0}$, $\boldsymbol{L_0}$ - system size, $\delta_i = mc/\sqrt{4\pi e^2 \rho_0}$

$\boldsymbol{V_0}$ and $\boldsymbol{\rho_0}$ are the representative flow velocity and mass density in the disk.

Stationary Solutions $\partial_t = 0$

To derive the term that balances the viscosity term we decompose the "inertia term"

$$\rho = \rho_1 \rho_2 \quad ===> \quad \boldsymbol{P}_1 := \rho_1 \boldsymbol{V}, \quad \boldsymbol{P}_2 := \rho_2 \boldsymbol{V}. \tag{18}$$

and we write:

$$\nabla \cdot (\boldsymbol{V}\boldsymbol{P}) = \nabla \cdot (\rho \boldsymbol{V}\boldsymbol{V}) = (\nabla \cdot \boldsymbol{P}_1)\boldsymbol{P}_2 + (\boldsymbol{P}_1 \cdot \nabla)\boldsymbol{P}_2.$$

Present analysis **– we choose a different separation of "inertia term" from conventional Fluid mechanics to match** $(\nabla \cdot \boldsymbol{P}_1)\boldsymbol{P}_2$ **with viscosity term.**

By *reduced* ρ_2 **(< 1) we will define *a* *generalized vorticity* of a *reduced momentum*.**

After straightforward algebra, **assuming a barotropic relation** $\nabla p = \rho \nabla h$ with an enthalpy ***h*** we obtain**:**

$$\boldsymbol{P}_2 \times \boldsymbol{\Omega}_2 = \frac{1}{2}\nabla P_2^2 + \rho_2^2 \nabla(\phi + h) + \frac{\rho_2}{\rho_1}\left[(\nabla \cdot \boldsymbol{P}_1)\boldsymbol{P}_2 + \nabla \cdot \boldsymbol{\Pi}\right], \tag{19}$$

Where

$$\boldsymbol{\Omega}_2 := \nabla \times \boldsymbol{P}_2 \tag{20}$$

Beltrami model of a Disk-Jet System

In the Disk flow $\boldsymbol{V}$ is primarily azimuthal **(** $\boldsymbol{\theta}$ **),** the viscosity force can be approximated as**:**

$$-\nabla\cdot\mathbf{\Pi}\approx-\nabla\times(\rho\eta\nabla\times\boldsymbol{V}),$$

In a Keplerian thin disk $V\approx V_0\, r^{-1/2}\, e_\theta$ **and** $\nabla(\rho\eta)$ **is approximately vertical,**

***we estimate*:**

$$-\nabla\cdot\mathbf{\Pi}\approx-\rho\eta\nabla\times(\nabla\times\boldsymbol{V})=-\rho\eta V_0\frac{3}{4}r^{-5/2}\boldsymbol{e}_\theta. \qquad (21)$$

hence, we may write

$$-\nabla\cdot\mathbf{\Pi}=-\nu\boldsymbol{P}$$ **with** $\nu > 0$ **===>**

viscosity force is primarily in azimuthal (toroidal) direction , it can be balanced by term

$$(\rho_2/\rho_1)(\nabla\cdot\boldsymbol{P}_1)\boldsymbol{P}_2$$ **extracted from the inertia term!**

Using steady state Mass Conservation Law we observe that **the balance of the viscosity and the partial inertia term demands**

$$\boldsymbol{V}\cdot\nabla\log\rho_2=\nu, \qquad (22)$$

which determines $\boldsymbol{\rho_2}$ **;** the straightforward algebra gives the relation for $\boldsymbol{\rho_1}$

$$\boldsymbol{V}\cdot\nabla\log\rho_1=-\nabla\cdot\boldsymbol{V}-\nu. \qquad (23)$$

The vorticity $\boldsymbol{\Omega_2}$ includes a singular factor $\nabla\times\boldsymbol{V}\propto r^{-3/2}\boldsymbol{e}_z$

To eliminate divergence of $\boldsymbol{P}_2\times\boldsymbol{\Omega}_2$ near the axis $\boldsymbol{P_2}$ *must allign* $\boldsymbol{\Omega_2}$, i.e.

Beltrami Condition **must be satisfied** (λ is a certain scalar function)

$$\boldsymbol{\Omega}_2=\lambda\boldsymbol{P}_2 \tag{24}$$

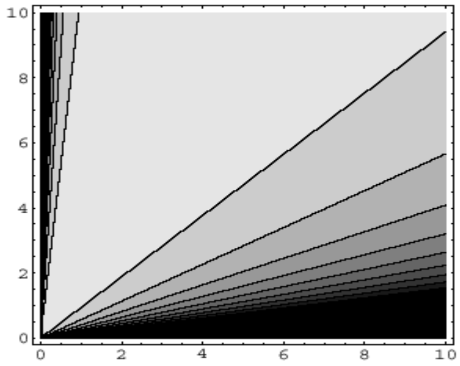
The flow $\boldsymbol{V}=\boldsymbol{P}_2/\rho_2$ is, therefore, collimated by the generalized vorticity $\boldsymbol{\Omega_2}$ creating a jet!

The remaining potential forces in (19) must balance ===> *Bernoulli Condition*

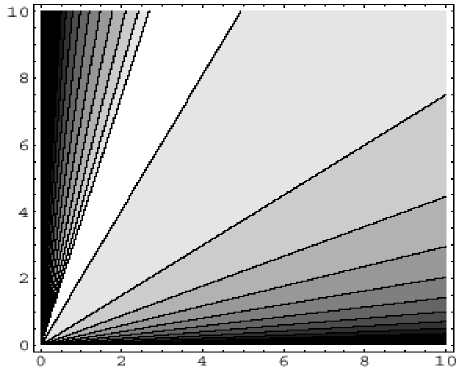
$$\frac{1}{2\rho_2^2}\nabla P_2^2+\nabla(\phi+h)=\nabla\left(\frac{1}{2}V^2+\phi+h\right)+V^2\nabla\log\rho_2=0. \tag{25}$$

The determining equations are: *Eq*. (22) determines “artificial ingredient” $\boldsymbol{\rho_2}$ for given $\boldsymbol{v}$; $\boldsymbol{V}(=\boldsymbol{P}_2/\rho_2)$ is governed by (24); after determining $\boldsymbol{V}$ and $\boldsymbol{\rho_2}$ we solve (25) to determine $\boldsymbol{h}$.

A similarity solution modeling fundamental disk-jet structure

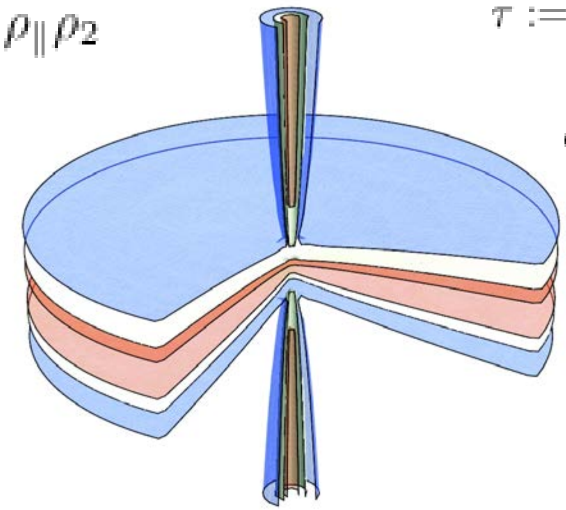


The momentum field (streamlines of poloidal component of *P*) of the similarity solution



The distribution of $\rho_\perp$ of the similarity solution,

$$\rho = \rho_\perp \rho_\parallel \rho_2$$



$$\tau := \frac{z}{r} \quad (r > 0),$$

$$\sigma := \sqrt{r^2 + z^2}\,.$$

$$(\nabla\tau \cdot \nabla\sigma = 0)$$

The density ρ in the similarity solution (*log* scale)

We assume $\rho_{||}(\sigma) \approx z^{-2/3}$; **and** $\rho_2 \approx r^{-1/2}$.

A levelset surface of ρ is shown in the domain $r < 5$ $|z| < 5.$

Summary for Disk-Jet Problem

Invoking the simplest (minimum) model of MHD we have shown that:

- **the combination of a thin disk and narrowly-collimated jet is the unique structure that is amenable to the singularity of the Keplerian vorticity - *the Beltrami condition* - the alignment of flow and generalized vorticity - characterizes the geometry.**

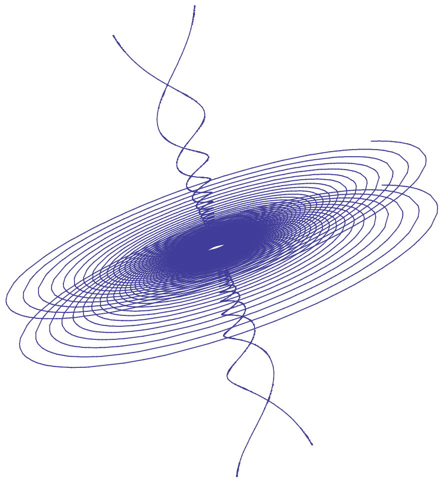
- **The conventional vorticity is generalized to to subtract the viscosity force causing the accretion and the centrifugal force of the Keplerian velocity.**

- **We have found an analytic solution in which the *generalized vorticity* is purely kinematic ($\Omega = \nabla \times P_2$ with the momentum P_2 modified by the viscosity effect).**

- We described a pure fluid-mechanical model of jet collimation, **magnetic field**, thrusting the center of the disk, **to "guide"** (*and twist, as often observed*) **the flow of charged gas** (plasma), **can be easily invoked**. **Here the *fluid generalized vorticity* plays the same role of *a magnetic field*.**

- **In charged disk additional magnetic force may contribute to jet acceleration if the self-consistently generated large-scale magnetic field is sufficiently large – such structures can be described by the generalized model.**

The Generalized Beltrami Vortex



The streamlines of "generalized Beltrami Flow" – in a disk-jet system the accreting flow and jet align parallel to *a generalized vorticity*

Identifying the disk-jet structure as a generalized Beltrami vortex, we will be able to understand the self-organization process in terms of the "generalized helicity" ==>

Important lesson **- the *helicity* of the *generalized vorticity* is the key parameter that characterizes the self-organizing of a disk-jet system.**

THANK YOU !