ON THE APPROXIMATE SOLUTION OF THE ONE DYNAMIC NONLINEAR PROBLEM FOR A PLATE

Vladimer Odisharia, Jemal Peradze

e-mail: vladimer.odisharia@tsu..ge, jemal.peradze@tsu.ge

Inter-discipline (Mathematics, Computer Sciences): Math. Logic and Discrete Structures,

Iv. Javakhishvili Tbilisi State University, Tbilisi, Georgia

By using the approach due to Berger [1] it was shown by Wah [3] that the vibration of rectangular plates $\Omega=\{(x,y)\mid \ 0<x<a,\ 0<y<b\}$ with large amplitudes may be described by the nonlinear differential equation

$$\frac{\partial^2 w}{\partial t^2}+\alpha\,\Delta^2 w-\beta\left[\int_\Omega\left(\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right)dx\,dy\right]\Delta w=0, \qquad (1)$$

in which $w(x,y,z)$ is lateral deflection and α and β are some nonnegative constants.

Consider equation (1) under the following initial boundary conditions

$$\frac{\partial^p}{\partial t^p}w(x,y,0)=w^p(x,y), \quad p=0,1, \quad w(x,y,t)\big|_{\partial\Omega}=0, \qquad (2)$$

where $w^0(x,y)$ and $w^1(x,y)$ are the given functions, $\partial\Omega$ is the boundary of the domain Ω.

Let us perform approximation of the solution of problem (1), (2) with respect to the variables x and y. For this, we use the Galerkin method. A solution will be sought in the form of the series

$$w_{mn}(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}w_{ij}^{mn}(t)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b},$$

where the coefficients $w_{ij}^{mn}(t)$ are the solution of the system of differential equations

$$\left(w_{ij}^{mn}(t)\right)''+\alpha\left(\left(\frac{\pi i}{a}\right)^2+\left(\frac{\pi j}{b}\right)^2\right)^2 w_{ij}^{mn}(t)+$$

$$+\frac{1}{4}ab\beta\left(\sum_{k=1}^{m}\sum_{l=1}^{n}\left(\left(\frac{\pi k}{a}\right)^2+\left(\frac{\pi l}{b}\right)^2\right)w_{kl}^{mn}(t)\right)\left(\left(\frac{\pi i}{a}\right)^2+\left(\frac{\pi j}{b}\right)^2\right)w_{ij}^{mn}(t)=0,$$

$$i=1,2,\dots,m, \qquad j=1,2,\dots,n,$$

with the initial conditions

$$\frac{d^p}{dt^p}w_{ij}^{mn}(0)=\frac{4}{ab}\int_\Omega w^p(x,y)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\,dx\,dy, \qquad p=0,1.$$

Applying the technique developed in [2] for a one-dimensional problem, we estimate the error of the Galerkin method.

References

1. H.M Berger, A new approach to the analysis of large deflections of plates, J. Appl. Mech., 22, no. 4, 465-472, 1955
2. J. Peradze, On the accuracy of the Galerkin method for a nonlinear beam equation, Math. Meth. Appl. Sci., 34, 1725-1732, 2011
3. T. Wah, Large amplitude flexural vibration of rectangular plates, Int. J. Mech. Sci., v. 3, no. 6, 425-438, 1963