Extremal problems and asymptotic estimations

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The questions discussed in this paper historically is connected to Lebesgue theorem, which concerns to the estimation of the deviation from the continuous function to partial sums of trigonometric Fourier series, namely

$$| f(x) - S_n(f;x) | \leq (\ln n + 3) E_n(f)$$

where $E_n(f)$ is the best approximation of the function f by trigonometric polynomials T_n of the degree n i.e.

$$E_n(f) = \inf_{T_n} \max_{x} | f(x) - T_n(x) | .$$

Because of the estimations obtained of the $E_n(f)$, Lebesgue's mentioned inequality does not loose actuality and is convenient for the estimations.

The results was generalized, namely, considered quantity:

$$\zeta(H;S_n) = \sup_{f \in H} \| f(x) - S_n(f;x) \|$$

where H is a some functional space. For different H spaces there is obtained asymptotic estimations of the quantity $\zeta(H;S_n)$.

In the paper reviewed asymptotic estimations of the quantity $\zeta(H;S_n)$ for different H spaces which was obtained by Kolmogorov, Nikolski, Stepanets and Kornechuk.

Defined the ω module of continuity and considered elementary extremal problem for the functions from H_ω space, its applications to estimate asymptotic behavior of the $\zeta(H_\omega;S_n)$ and estimation of the upper bound of Fourier coefficients of functions which belongs to the H_ω space.