On the invariants of Fuchsian system

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Consider a family of Fuchsian systems

$$\frac{df}{dz} = \left(\sum_{j=1}^{n} \frac{B_j(s)}{z-s_j}\right) f, \ \sum_{j=1}^{n} B_j(a) = 0 \tag{1}$$

depending holomorphically on the parameter $s = (s_1, s_2, \dots, s_n) \in D(s^0)$, where $D(s^0)$ ia a small disk with the center $s^0 = (s_1^0, s_2^0, .., s_n^0)$ in the space $\mathbf{C}^n \backslash \cup_{i \neq j} \{(s_i - s_j) = 0\}$. Family (1) is considered on the space $T = \mathbf{CP}^1 \times D(s^0) \backslash \cup_{i \neq j} \{(s_i - s_j) = 0\}$ wich can be retracted to $\mathbf{C}\mathrm{P}^1 \backslash \{s_1^0, s_2^0, \dots, s_n^0\}$. Therefore the fundamental group $\pi_1(T, (z_0, s^0))$ is isomorphic to the group $\pi_1(\mathbf{CP}^1 \backslash \{s_1^0, s_2^0, \dots, s_n^0\}, \mathrm{z}_0)$ which, in turn, is generated by the homotopic classes of loops γ_j^0 going around the singular points s_j^0 along small circles respectively. For any isomonodromic family (1) there exists an isomonodromic family of fundamental matrices analytic both in z and s in T.

Proposition. For the fundamental matrix $F(z,s)$ of familly (1) the following decomposition holds in a neighboorhood of $\{z - s_j = 0\}$:

$$F(z,s)S(a) = U_j(z,s)(z - s_j)^{D_j}(z - s_j)^{E_j(a)}, \tag{2}$$

where $S(a)$ is holomorphically invertible in $D(s^0)$ and has the same block diagonal structure as E_j, $U_j(z,s)$ is a holomorphically invertible at $\{z - s_j = 0\}$, D_j has the same block diagonal structure $D_j = diag(D_j^1, \dots, D_j^k)$ as E_j with diagonal integer-valued matrices D_j^l whose entries satisfy certain inequalities, and $E_j(s) = S^{-1}(s) E_i S(s)$.

Proposition. Every isomonodromic deformation preserves the eigenvalues $\{b_j^l\}$ of the coefficient matrices $B_j(a)$ from (1) and entries $\{d_j^l\}$ of the matrix D_j from (2).

From this propositions follows, that isomonodromic deformation don't change the holomorphic structures corresponding vector bundles induced from given Fuchsian system (more detailed analysis see [1],[2]).

References

[1] G.Giorgadze, Moduli space of complex structures. J. Math.Sci.(N.Y.), 160, **6**, 2009, 697-716.
[2] G.Giorgadze, G.Khimshiashvili, The Riemann-Hilbert problem in loop spaces. Doklady Mathematics, 73, **2**, 2006, 258-260.