Boundedness of Stein's spherical maximal function in variable Lebesgue spaces and application to the wave equation

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Define the spherical maximal operator M_σ, by

$$M_\sigma(f)(x) = \sup_{t>0} \left| \int_{S^{n-1}} f(x-t\theta)d\sigma(\theta) \right|, \qquad f \in S(R^n),$$

where $d\sigma$ denotes the normalized Lebesgue measure on the unit sphere S^{n-1} of space R^n. The maximal operator M_σ is bounded on $L^p(R^n)$ if and only if $p > n/(n-1)$, and this range of p is optimal. E.M. Stein proved this in three or more dimensions, J. Burgain in two dimensions.

We prove that if $p: R^n \to (0,\infty)$ $n \geq 3$ is globally log-Holder continuous and its infimum p_- and its supremum p^+ are such that $\frac{n}{n-1} < p_- < p^+ \leq (n-1)p_-$, then the spherical maximal operator is bounded in $L^{p(\cdot)}(R^n)$. When $n=3$, the result is then interpreted as the preservation of the integrability properties of the intial velocity of propagation to the solution of the intial-value problem for the wave equation.