Reduction of a Three-Layer Semi-Discrete Scheme for an Abstract Parabolic Equation to Two-Layer Schemes. Explicit Estimates of the Approximate Solution Error

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Let us consider the following evolutionary problem in the Banach space X :

$$\frac{du}{dt} + Au(t) = 0, \quad t \in]0, T], \quad u(0) = u_0 , \tag{1}$$

where $(-A)$ is the generating operator of a strongly continuous semi-group; u_0 is a given vector from X ; $u(t)$ is the sought abstract function with values from X .

We introduce on $[0, T]$ the net $t_k = k\tau$, $k = 1, 2, \dots, n$, with pitch $\tau = T / n$. A purely implicit three-layer semi-discrete approximation scheme of second order is considered for an approximate solution of equation (1). Using the perturbation algorithm, this scheme is reduced to two two-layer schemes, one of which corresponds to the zero degree and the other to the first degree of a small parameter (here τ plays the role of a small parameter). The solutions of these schemes are respectively denoted by $u_k^{(0)}$ and $u_k^{(1)}$. Let the vector $v_k = u_k^{(0)} + \frac{\tau}{2} u_k^{(1)}$ be an approximate value of the exact solution of problem (1) for $t = t_k$, $u(t_k) \approx v_k$.

The following theorem is true.

Theorem. Let A be a linear, densely defined closed operator in the Banach space X. Assume that the sector $|\arg(z)| < \varphi$, $0 < \varphi < \pi / 2$, completely contains the spectrum of the operator A and for any $z \neq 0$ not belonging to this sector the condition $\|(zI - A)^{-1}\| \le c_0 |z|^{-1}$, $c_0 = const > 0$, is fulfilled. Then the estimate

$$\|u(t_k) - v_k\| \le c\tau^2 \ln\left(\frac{et_k}{\tau}\right) \|A^2 u_0\|, \quad k = 2, \dots, n,$$

is valied, where $u_0 \in D(A^2)$, the constant $c > 0$ does not depend on the solution of the initial problem.

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