On Compact Wavelet Matrices

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Representation of a positive definite matrix function $S(t)$, defined on the unit circle T, in a form of the product $S(t)=S_+(t)S_-(t)$, where S_+ is a regular analytic matrix function inside T and $S_-(t)$ is its adjoint, is called the spectral factorization. This factorization plays an important role in the solution of various practical problems in Control Theory and Communications. Consequently there exist several different methods of spectral factorization. In recent years it has been developed a general effective method of matrix spectral factorization [1], [2]. It turned out that this method is closely related with the theory of wavelets. The wavelets represent specific orthonormal bases of the Hilbert space $L_2(R)$ which theory, as a branch of Harmonic Analysis, has been actively developed since 90-ies of the last century. Wavelets naturally substitute the Fourier Analysis in solution of different engineering problems. As computer implementations of existing algorithms play an important role in the realization of these problems, it has been developed the discrete theory of wavelets as well in the form of wavelet matrices.

A main idea of the above mentioned new method of spectral factorization has been successfully used for complete parametrization of compact wavelet matrices (of rank m and order and degree N) in terms of $(m-1)N$ dimensional Euclidian spaces. A different paramtrization existed before in terms of more complex projective spaces, and the wavelet matrix generation and completion algorithms were based on this parametrization. The new parametrization enable us to develop a different viewpoint of both these algorithms and propose their more effective computer implementations [3]. The essence of the new algorithms consists in a one-to-one map between the corresponding to wavelet matrices unitary matrix polynomials $U(t)$ and their Wiener-Hopf factors $U_-(t)$, where $U(t)=U_-(t)U_+(t)$. In turn, the matrices $U_-(t)$ are described in a simple form. It is written explicitly how to construct $U_-(t)$ from $U_-(t)$ and vice versa. These results in details are given in preprint [4].

ლიტერატურა

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