ON THE ESTIMATION OF A MAXIMUM LIKELIYHOOD OF TRUNCATED EXPONENTIAL DISTRIBUTIONS

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Let X be a truncated exponentially distributed random value with density

$$f(x;\theta,\alpha,\beta)=\begin{cases}\dfrac{\theta e^{-\theta x}}{e^{-\alpha\theta}-e^{-\beta\theta}}, & \alpha<x\le\beta\\ 0, & x\le\alpha,\ x>\beta\end{cases}, \qquad (1)$$

where $\alpha<\beta$ and α, β, and θ are the unknown parameters. Let $X_1,X_2,\dots,X_n$ be a random sampling of size n taken from the truncated exponential distributions given by (1). It is required to estimate α, β and θ by these observations. We do this by applying the maximum likelihood estimator.

A likelihood function has the form

$$L(x;\theta,\alpha,\beta)=\theta^n\left(e^{-\alpha\theta}-e^{-\beta\theta}\right)^{-n}\cdot\exp\left(-\theta\sum_{i=1}^{n}x_i\right)=\theta^n\left(e^{-\alpha\theta}-e^{-\beta\theta}\right)^{-n}\cdot\exp\left(-n\theta\bar{X}\right). \qquad (2)$$

Theorem. *Assume that we have the sample $X_1,X_2,\dots,X_n$ of random values which are distributed according to law* (1) *where α, β and θ are the unknown parameters.*

If $0<\bar{X}<\dfrac{\beta+\alpha}{2}$, then the maximum likelihood estimate for θ exists and is the unique root of the equation

$$\frac{1}{\theta}-\left(\beta e^{-\beta\theta}-\alpha e^{-\alpha\theta}\right)\left(e^{-\alpha\theta}-e^{-\beta\theta}\right)^{-1}-\bar{X}=0. \qquad (3)$$

This estimate is consistent and asymptotically effective.

Also, $\alpha=X_{(1)}=\min(X_1,\dots,X_n)$, $\beta=X_{(n)}=\max(X_1,\dots,X_n)$.

Remark. If $\alpha=0$ and $\beta=\infty$, then from (3) we obtain the classical case

$$\theta=\frac{1}{\frac{1}{n}\sum_{i=1}^{n}x_i}.$$

Computerized simulation of an exponential distribution with parameters $\alpha=1$, $\beta=2$ and $\theta=2$ was carried out. For the sample of size $n=10000$, we obtained the estimate $\bar{X}=1{,}344$, $\hat{\theta}=2{,}005$.

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