TENSOR INVARIANTS AND HOMOGENEOUS RIEMANN SPACES

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Let $M = \wp / \hbar$ is a homogeneous Riemann space where $\hbar$ is a compact semi-group of the Lie group $\wp$. Manturov O. V. and J.A. Wolf gives the classification for homogeneous Riemann spaces with a stationary point group acting irredicibly on a tangential space - these are the so-called homogeneous Riemann spaces with an irreducible isotropy group [1], [2].

We have the following expansion at a direct sum:

$$G = H + B,$$

where H and G are the compact Lie algebras corresponding to the Lie groups $\hbar$ and $\wp$, respectively, and B is a linear subspace of the G, which is invariant with respect to the transformation $Ad_h()$, $h \in H$, on the G. Linear space B called the algebra isotropy of the homogeneous space M. Any invariant tensor with respect to the algebra of isotropy B generates invariant tensor field on the homogeneous space.

For the homogeneous space considered here, groups of isotropy are linear groups given by some irreducible isotropic representations φ of the Lie algebra H, of the corresponding homogeneous Riemann space $M = \wp / \hbar$.

We consider the tensor square of an irreducible isotropic representation φ and makes it expands into a direct sum of irreducible components:

$$\varphi \otimes \varphi = \sum_{i=1}^{p} k_{\varphi_i} \varphi_i \quad ,$$

where φ_i are the irreducible representation of the Lie algebra H and k_{φ_i} are their multiplicities. Using the Shur's lemma, we calculate the dimensions of spaces of invariant tensor fields of valence 3 and 4.

Our main results is the following (to appear in "Journal of Mathematical Sciences", Springer. Journal no. 10958): calculating this dimensions when for the homogeneous spaces the subgroup $\hbar$ has the type of a simple algebra A_n, $n \geq 1$ and $A_1 + B_n$, $n \geq 2$.

References

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