On the almost everywhere summability of double series with respect to diagonal block-orthonormal systems

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The blockwise orthogonal and block-orthogonal systems were considered by Moricz and Gaposhkin. Gaposhkin proved that the coefficient test of Menshov-Rademacher ensuring the almost everywhere convergence of orthogonal series and the strong law of large numbers are valid for such systems in certain conditions. Further were obtained some results on convergence and summability of series with respect to block-orthonormal systems. In particular, Menshov-Rademacher's and Gaposhkin's theorems were generalized and the exact Weyl multipliers for the convergence and summability almost everywhere of series with respect to block-orthogonal systems were established by Nadibaidze in the case, when Menshov-Rademacher's and Gaposhkin's theorems are not true. Moricz proved two-dimensional analog of Kacmarz`s coefficient test ensuring the almost everywhere $(C,1,1)$, $(C,1,0)$ and $(C,0,1)$ summability of double orthogonal series

Below some questions connected with the problems of almost everywhere summability of double series with respect to diagonal block-orthonormal systems are considered.

Let $\{M_k\}$ and $\{N_k\}$ be increasing sequences of natural numbers and

$$\Delta_k = ([1, M_{k+1}] \times [1, N_{k+1}]) \setminus ([1, M_k] \times [1, N_k]), \quad (k \geq 1) .$$

Let $\{\varphi_{mn}\}$ be a system of functions from $L^2((0,1)^2)$. The system $\{\varphi_{mn}\}$ will be called a diagonal Δ_k-orthonormal system if $\|\varphi_{mn}\|_2 = 1$, $m = 1,2,..., n = 1,2,...$ and $(\varphi_{ij}, \varphi_{pq}) = 0$, for $(i, j),(p,q) \in \Delta_k$, $(i, j) \neq (p,q)$, $(k \geq 1)$.

The extension of the coefficient test of Menshov and Kaczmarz ensuring the almost everywhere $(C,1,1)$, $(C,1,0)$ and $(C,0,1)$ summability of double series with respect to diagonal block-orthonormal systems is studied. The necessary and sufficient conditions on the length of blocks are obtained for which two-dimensional analog of Kacmarz`s theorem is valid for the block-orthogonal series. Furthermore, it is established the conditions connected with the relation between block's length and Weyl multipliers for the almost everywhere summability of double series with respect to diagonal block-orthonormal systems. Using this conditions it is possible determine exact Weyl multipliers for the Cesaro $(C,1,1)$, $(C,1,0)$ and $(C,0,1)$ summability of double series with respect to diagonal block-orthonormal systems in that case, when the two-dimensional analog of Kacmarz`s theorem is not true.

Finally, it is established the minimal order of block's growth for which the exact Weyl multipliers for the summability almost everywhere of double orthogonal serie are valid for the series with respect to diagonal block-orthonormal systems.