ON THE BOUNDED QUASI-DEGREES OF C.E. SETS

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Tennenbaum (see [1,p.159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is Q-reducible to a set B (in symbols: $A\leq_Q B$) if there exists a computable function f such that for every $x\in\omega$ (where ω denotes the set of natural numbers),

$$x\in A \Leftrightarrow W_{f(x)}\subseteq B.$$

In this case we say that $A\leq_Q B$ via f.

If $A\leq_Q B$ via f and there exists a fixed number $n\in\omega$ such that for all x, $|W_{f(x)}| \leq n$, then we say that the set A is bounded Q-reducible to the set B, denoted by $A\leq_{bQ} B$. A computably enumerable (c.e.) set A is bQ-complete if every c.e. set is bQ-reducible to A. The relation of bQ-reducibility is reflexive and transitive so that it generates a degree structure on the subsets of ω.

Our notation and terminology are standart and can be found in [1].

Theorem 1. For every noncomputable c.e. incomplete bQ-degree, there exists a nonspeedable bQ-degree incomparable with it.

Corollary . Among all nonspeedable bQ-degrees contained in a c.e. complete T-degree there is no maximal bQ-degree.

A noncomputable set X is called computable separable (shortly: r-separable), if for every c.e. set Y

$$Y\cap X=\emptyset \Longrightarrow (\exists R \text{ computable})(X\subseteq R \,\&\, Y\cap R=\emptyset).$$

Theorem 2. Let A and B be c.e. sets and suppose that A is an r-separable set. Then if $A\leq_{bQ} B$, then there exists a noncomputable c.e. C such that $C\leq_m A$ & $C \leq_m B$.

Corollary. Let M be a maximal set and A a c.e. set. Then $M \leq_{bQ} A \Longrightarrow M\leq_m A$.

Theorem 3. The structure $\mathfrak{D}_{bs}$ of the bs-degrees is not elementary equivalent neither to the structure $\mathfrak{D}_{be}$ of the be-degrees nor to the structure $\mathfrak{D}_e$ of the e-degrees.

Corollary. The structure $\mathfrak{D}_{bQ}$ of the bQ-degrees is not elementary equivalent neither to the structure $\mathfrak{D}_{be}$ of the be-degrees nor to the structure $\mathfrak{D}_e$ of the e-degrees.

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[1] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.