On the summability of Fourier series by the generalized Cesáro means

Teimuraz Akhobadze
e–mail: teimuraz.akhobadze@tsu.ge
Departament of Mathematics, Ivane Javakhishvili Tbilisi State Universite,
Chavchavadze str.1, Tbilisi 0128 Georgia

Let (α_n) be a sequence of real numbers where $\alpha_n > -1,\ n = 1, 2, \ldots$. Suppose

$$\sigma_n^{\alpha_n}(x, f) =: \sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n - 1} S_\nu(x, f) / A_n^{\alpha_n} ,$$

where

$$A_k^{\alpha_n} = (\alpha_n + 1)(\alpha_n + 2)\ldots(\alpha_n + k) / k!.$$

If (α_n) is a constant sequence $(\alpha_n = \alpha,\ n = 1, 2, \ldots)$ then $\sigma_n^{\alpha_n}(x, f)$ coincides with the usual Cesáro $\sigma_n^{\alpha}(x, f)$ -means. In the case of convergence of means $\sigma_n^{\alpha_n}(x, f)$ we say that the Fourier series of f is (C, α_n)–summable.

One of the most general test of convergence of Fourier series at a point was given by Lebesgue. Gergen improved the Lebesgue statement. Zhizhiashvili proved analogous of the Gergen theorem for (C, α) means $(-1 < \alpha < 1)$ of trigonometric Fourier series.

The purpose of our report is to generalize Zhizhiashvili's statement for (C, α_n)–summability.

Theorem. Let $-1 < \alpha_n < 1,\ n = 1, 2, \ldots,\ \varphi(x,t) = f(x+t) + f(x-t) - 2f(x)$ and

$$\overline{\Phi}(x,t) = \sup_{0 \le u \le t} |\Phi(x,t)|.$$

Supposed, that

$$\text{(1)} \qquad \frac{1}{(1+\alpha_n)n} \int_{\frac{\pi}{n}}^{\pi} \frac{\overline{\Phi}(x,t)}{t^3} dt = o(1)$$

and

$$\text{(2)} \qquad \frac{1}{(1+\alpha_n)n^{\alpha_n}} \sup_{0 < h \le \frac{\pi}{n}} \int_{\frac{\pi}{n}}^{\pi} t^{-1+\alpha_n} |\varphi(x,t) - \varphi(x,t+h)| dt = o(1),\ n \to \infty.$$

Then the trigonometric Fourier series is (C, α_n)–summable at x. Summability is uniform over any closed interval of continuity where (1) and (2) are satisfied uniformly.