Logarithmic Sobolev type inequalities for Poisson functionals

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For the Gaussian measure ν on R^n is well known the so called logarithmic Sobolev inequality:

$$\int_{R^n} | f(x)|^2 \ln | f(x)| d\nu(x) \leq \int_{R^n} | grad f(x)|^2 \, d\nu(x) + \| f \|_2^2 \ln \| f \|_2$$

where $\| f \|_p$ denotes the $L_P(\nu)$ norm of f. Logarithmic Sobolev (or Log-Sobolev) inequalities were introduced by L. Gross in 1975 as a way of isolating smoothing properties of Markov semigroups in infinite-dimensional settings. It can be used to obtain quantitative bounds on the convergence of finite Markov chains to stationary. Log-Sobolev inequalities are one of the essential tools for proving concentration phenomena, not only because they require in some sense less understanding about the underlying geometry of the measured space, but also because they yield sharper results for concentration, i.e., Gaussian rather than exponential. They are particularly well-suited for infinite-dimensional analysis.

Using the Clark-Haussmann-Ocone representations for compensated Poisson functionals, which was proved by us based on the explicit constructions of the stochastic derivative operator for compensated Poisson functionals, we prove the Logarithmic Sobolev type inequalities for stochastic differentiable square integrable Poisson functionals. We verifying also the Cauchy-Bunyakovsky type inequality for the conditional mathematical expectation.