Sobolev type inequalities in infinite dimensional space

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In mathematical analysis a class of Sobolev inequalities, relating norms including those of Sobolev spaces. These are used the Sobolev embedding theorem, giving inclusions between certain Sobolev spaces, and the Rellich Kondrachov theorem showing that under slightly stronger conditions some Sobolev spaces are compactly embedded in others. Because, Sobolev inequalities relative the size of ∇f to the size of f, in order to prove such inequalities, one may try to express f in terms of its gradient. There are several ways to approach the notion of derivative $f^{'}(x)$. The first notion of a derivative of a function on a vector space is that of the Frechet derivative. The second and weaker notion of derivative is the Gateaux derivative. But these two notions of differentiability are too strong for many purposes. Many pathologies arise when dealing with infinite dimensional spaces that are not present with finite dimensional ones. If we suppose that Banach space E supports a Gaussian measure μ, and $H_{\mu} \subset E$ be the associated reproducing kernel Hilbert space, then the Malliavin calculus concerns functions on E that are differentiable in the directions of H_{μ}. It turns out that a function may be differentiable in this weak sense, and yet not even be continuous on E!

The Malliavin derivative is a linear map from a space of random variables to a space of processses indexed be a Hilbert space. Being a derivative, it is not surprising that this operator is unbounded. If the random variable ξ is differentiable (in Malliavin sense), the Clark-Ocone formula allows one to explicitly compute the integrand in the martingale representation in terms of the Malliavin derivative of ξ. In turn, the Clark-Ocone formula allow one to prove the Sobolev type inequalities in Wiener case. A further generalization of Clark-Ocone formula for $\xi \in D_{1,2}^{M}$ (which called the Clark-Haussmann-Ocone representation) belongs to Ma, Protter and Martin for the so-called normal martingales classes. We introduce the Sobolev type spaces $D_{1,q}^{M}$, where $1 < q < 2$, and generalize the Clark-Haussmann-Ocone representation for functional from these spaces. On the other hand, we receive the Clark-Haussmann-Ocone explicit formula in Poisson cases and using the above-mentioned representations, prove the Sobolev type inequalities on the one hand in general case for a class of normal martingales and on the other hand to give more explicit estimations in special case for Poisson functionals.