ON THE CRAMER-RAO INEQUALITY IN AN INFINITE DIMENSIONAL SPACE

SOKHADZE GRIGOL [a]

NADARAYA ELIZBAR [b], PURTUKHIA OMAR [c]

e-mail: grigol.sokhadze@tsu.ge

[a,b,c] Depattment of Mathematics, Faculty of Exact and Natural Sciences,

Iv. Javakhishvili Tbilisi State University,

2 University Str., Tbilisi 0186, Georgia

Let, $\{\aleph, \Re, (P(\theta;\cdot), \theta \in \Theta)\}$ is a statistical structure, where $\aleph$ separable, real Banach space, and Θ smooth manifold immersed in another separable, real Banach space Ξ. For every fixed $A \in \Re$ and a vector $\vartheta \in \Xi$ consider derivative of the function $\tau(\theta) = P(\theta; A)$ in a point θ along of ϑ. Denote this derivative by $d_\theta P(\theta; A)\vartheta$. For a fixed θ and ϑ it is the sign-changing measure and $d_\theta P(\theta, \cdot)\vartheta << P(\theta, \cdot)$. Than $l_\theta(x; \vartheta) = \dfrac{d_\theta P(\theta; dx)\vartheta}{P(\theta; dx)}$ is called a logarithmic derivative of the measure $P(\theta; \cdot)$ on parameter. Let $z(x): \aleph \to \aleph$ be a differentiable vector field possessing the bounded derivative. Let $\beta_\theta(x; z(x))$ is logarithmic derivative of $P(\theta; \cdot)$ along $z(x)$.

Lemma. Under regularity conditions for logarithmic derivatives $\beta_\theta(x; h)$ and $l_\theta(x; \vartheta)$ following equality is true $l_\theta(x; \vartheta) = -\beta_\theta(x; K_{\theta,\vartheta}(x))$, where

$$K_{\theta,\vartheta}(x) = E\left\{\frac{d}{d\theta} X(\theta)\vartheta \mid X(\theta) = x\right\}.$$

Let $\{\aleph, \Re, (P(\theta, \cdot), \theta \in \Theta)\}$ be a statistical structure corresponding to random element $X(\omega) = X(\theta, \omega)$. Here $\aleph$ is separable, real, reflective Banach space, $\Re$ - σ-algebra of the Borel sets. $\Theta \subset \Xi$ is open subset of separable, real Banach space Ξ. Let's assume that conditions of a regularity are fulfilled.

Suppose that $g(\theta) = E_\theta(T(X))$, where $T: \aleph \to R$ is measurable mapping (statistics). Concerning statistics we shall accept one more condition of a regularity.

Theorem (Cramer-Rao inequality). Let's assume that conditions of a regularity I-V are fulfilled. Then

$$VarT(X) \geq \frac{(g'_\vartheta(\theta))^2}{E_\theta \beta_\theta^2(X; E(X'(\theta)\vartheta \mid X))}.$$