

ON THE INTEGRAL SQUARE DEVIATION OF A ONE NONPARAMETRIC ESTIMATOR OF THE BERNOULLI REGRESSION FUNCTION

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Let is given sample $Y_j, j = \overline{1, n}, P(Y_j = 1 | x_j) = p(x_j), P(Y_j = 0 | x_j) = 1 - p(x_j), x_j = (2j - 1)/2n$, where $p(x)$ is unknown Bernoulli regression function. For estimating $p(x)$ is considered kernel type estimator:

$$\hat{p}_n(x) = p_{1n}(x) \cdot p_{2n}^{-1}(x), \quad p_{\nu n}(x) = \frac{1}{nb_n} \sum_{j=1}^n Y_j^{2-\nu} K\left(\frac{x - x_j}{b_n}\right), \quad \nu = 1, 2,$$

where $K(x)$ satisfies certain conditions and $b_n > 0, b_n \rightarrow 0$.

Is solved

Problem (this problem is exist from 70 years):

$$b_n^{-1/2} (T_n - \Delta) \sigma^{-1} \xrightarrow{d} N(0, 1),$$

where

$$\begin{aligned} T_n &= nb_n \int_{\Omega_n} [\hat{p}_n(x) - p(x)]^2 p_{2n}^2(x) dx, \quad \Omega_n = [\tau b_n, 1 - \tau b_n], \\ \Delta(p) &= \int_0^1 \Pi(x) dx \int_{|x| \leq \tau} K^2(x) dx, \quad \Pi(x) = p(x)(1 - p(x)), \\ \sigma^2(p) &= 2 \int_0^1 \Pi^2(x) dx \int_{|x| \leq 2\tau} K_0^2(x) dx, \quad K_0 = K * K. \end{aligned}$$

The obtained result allows us to construct a criterion for testing Hypothesis $H_0: p(x) = p_0(x)$. The question as to its consistency is studied. The power asymptotics of the constructed test is also studeid for Pitmans type of close alternatives.