Uniqueness Theorem of Solution of the Dirichlet BVP for Special Type High Order Elliptic Differential Equation in the Half-Space

Ilia Tavkhelidze [a] *, Maria Transirico* [b]
E-mail: ilia.tavkhelidze@rsu.ge [a], mtransirico@unisa.it
[a] Department of Mathematics, Iv.Javakhishvili State University, University str # 2, Tbilisi 0186 Georgia
[b] Department of Mathematics, Faculty of Sciences of University of Salerno, Via Ponte don Melillo, 84082, Fisciano, Italy

In the present report higher order, divergent type, elliptic differential equation, with variable and infinity-smooth coefficients

$$\sum_{|\alpha|=m} D^{\alpha}\left(a_{\alpha}(x)D^{\alpha}u(x)\right)= f(x)$$

is considered in the half-space $R_{+}^{n} \equiv \{x=(x_1,...,x_n): x_n > 0\}$. The uniqueness theorem for generalized solution of Dirichlet BVP for homogenous equation

$$\left.\frac{\partial^{j}u(x)}{\partial x_n^{j}}\right|_{x_n=0} = 0, \qquad j=\overline{0,..,m-1}$$

when "energy integral" is bounded

$$\int_{R_{+}^{n}} \sum_{|\alpha|=m} \left[D^{\alpha}u(x)\right]^2 < \infty$$

is proved. The process of proving is based on the generalized Hardy's inequality (see. [1]) and so theorem is hold when dimension of the space: a.) is bigger than order of equation (n>2m) or b.) is odd and less than order of equation (n=2k+1<2m). Method of proving for similar result for the biharmonic equation is given in the article [2].

References:

[1] V.A.Kondrat'ev and O.A.Oleinik, *Russian Math. Surveyes 43 (1988) #5 , 65 .*
[2] I.Tavkhelidze, *Georgian Mathematical Journal 14 (2007) 3, 565 .*