ON THE BASIS OF SOME SPACES OF GENERALIZED THETA-SERIES

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Let $Q(X)$ be an integer positive definite quadratic form in an even number r of variables and let $P(X)$ is a spherical polynomial of order ν with respect to $Q(X)$ (see [1]) ,

$$\vartheta(\tau, P, Q) = \sum_{n \in Z^r} P(n) z^{Q(n)}$$

$z = e^{2\pi i\tau}$, $\quad Im\,\tau > 0$,

is the corresponding generalized r-fold theta-series (see [2], pp. 808, 855).

Let $T(\nu, Q) = \{\vartheta(\tau, P, Q)\}$ denote the vector space over C of generalized multiple theta-series. Gooding [1] calculated the dimension of the vector space T (ν, Q) for reduced binary quadratic forms Q. In [3] we obtained the upper bounds for the dimension of the vector space T (ν, Q) for diagonal and for some nondiagonal ternary and quaternary quadratic forms. Namely, for the quadratic form $Q_1 = b_{11}x_1^2 + b_{22}(x_2^2 + x_3^2 + x_4^2)$ we proved:

$$\dim T(\nu, Q_1) \leq g_1(\nu) = \begin{cases} \frac{1}{3}\left(\frac{\nu}{4}+1\right)\left(\frac{\nu}{4}+2\right) & if\ \ \nu \equiv 4\ or\ 8\ (mod\ 12), \\ \frac{\nu}{4}\left(\frac{\nu}{12}+1\right)+1 & if\ \ \nu \equiv 0\ (mod\ 12), \\ \frac{(\nu+2)(\nu+10)}{48} & if\ \ \nu \equiv 2\ or\ 10\ (mod\ 12), \\ 3\left(\frac{\nu+6}{12}\right)^2 & if\ \ \nu \equiv 6\ (mod\ 12). \end{cases}$$

For the quadratic form $Q_2 = b_{11}x_1^2 + b_{22}x_2^2 + b_{33}(x_3^2 + x_4^2)$ we have

$$\dim T(\nu, Q_2) \leq g_2(\nu) = \begin{cases} \left(\frac{\nu}{4}+1\right)^2 & if\ \ \nu \equiv 0\ (mod\ 4), \\ \frac{1}{4}\left(\frac{\nu}{2}+1\right)\left(\frac{\nu}{2}+3\right) & if\ \ \nu \equiv 2\ (mod\ 4). \end{cases}$$

We form the basis of the vector spaces T (6, Q_1), T (8, Q_1) and T (6, Q_2) and showed that, these dimensions are equal to the upper bounds for the dimensions of the vector spaces T (6, Q_1), T (8, Q_1) and T (6, Q_2).

References

1. F. Gooding, Modular forms arising from spherical polynomials and positive definite quadratic forms, J.Number Theory, 9 (1): 36–47, 1977.

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