Free and projective $LinGL^2$-algebras

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An algebra $(A, \vee, \wedge, \lozenge, -, 0,1)$ is said to be GL- algebra (or diagonalizable algebra) if $(A, \vee, \wedge, \lozenge, -, 0,1)$ is a Boolean algebra and the unary operation $\lozenge$ satis_es the following conditions:
(1) $\lozenge(a \vee b) = \lozenge(a) \vee \lozenge(b)$, (2) $\lozenge(0) = 0$, (3) $\lozenge(a) \leq \lozenge(a \wedge -\lozenge(a))$.

An algebra $(A, \vee, \wedge, \lozenge_1, \lozenge_2, -, 0,1)$ is said to be $LinGL^2$ -algebra if (1) $(A, \vee, \wedge, \lozenge_1, -, 0,1)$ is a GL-algebra, (2) $(A, \vee, \wedge, \lozenge_2, -, 0,1)$ is a GL-algebra and (3) $\Box_1^{\rho}\lozenge_2^{\rho}x = \lozenge_2^{\rho}x$, $\lozenge_2^{\rho}\Box_1^{\rho}x = \Box_1^{\rho}x$, $\Box_1^{\rho}x = \Box_1 x \wedge x$, $\lozenge_2^{\rho}x = \lozenge_2 x \vee x$, $\Box_1 x = -\lozenge_1 - x$.

A description of m-generated free $LinGL^2$ -algebras in the variety $\mathbf{LinGL^2}$, corresponding to bi-modal provability logic $LinGL^2$, is given. The characterization of projective $LinGL^2$ –algebra is given. It is shown that any m-generated $LinGL^2$-subalgebra of m-generated free $LinGL^2$-algebra is projective.