Quantum corrections to the classical model of the atom-field system

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The nonlinear-oscillating system in action-angle variables is characterized by the dependence of frequency of oscillation $\omega(I)$ on action I. Periodic perturbation is capable of realizing in the system a stable nonlinear resonance at which the action I adapts to the resonance condition $\omega(I_0) \approx \omega$, that is, "sticking" in the resonance frequency. For a particular physical problem there may be a case when $I >> \hbar$ is the classical quantity, whereas its correction $\Delta I \approx \hbar$ is the quantum quantity. Naturally, dynamics of ΔI is described by the quantum equation of motion. In particular, in the moderate nonlinearity approximation $\varepsilon << (d\omega/dI)(I/\omega) << 1/\varepsilon$, where ε is the small parameter, the description of quantum state is reduced to the solution of the Mathieu- Schr¨odinger equation. The state formed as a result of sticking in resonance is an eigenstate of the operator $\Delta\hat{I}$ that does not commute with the Hamiltonian $\hat{H}$. Expanding the eigenstate wave functions in Hamiltonian eigenfunctions, one can obtain a probability distribution of energy level population. Thus, an inverse level population for times lower than the relaxation time can be obtained.