Free and projective MV(C)-algebras

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It is well known MV – algebras are algebraic models for Lukasiewicz logic L.

An algebra $A = (A; \otimes, \oplus, *, 0, 1)$ is said to be *an MV-algebra* iff it satisfies the following equations: 1) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$; 2) $x \oplus y = y \oplus x$; 3) $x \oplus 0 = x$; 4) $x \oplus 1 = 1$; 5) $0^* = 1$; 6) $1^* = 0$; 7) $x \otimes y = (x^* \oplus y^*)^*$; 8) $(x^* \oplus y)^* \oplus y = (y^* \oplus x)^* \oplus x$.

The unit interval of real numbers [0, 1] endowed with the following operations: $x \oplus y = \min(1, x + y)$, $x \otimes y = \max(0, x + y - 1)$, $x^* = 1 - x$, becomes an MV-algebra. For $(0 \neq)m \in \omega$ we set $S_m = (\{0,1/m, \ldots , m\text{-}1/m, 1\},\oplus,\otimes, *,0, 1)$.

An algebra $A = (A; \otimes, \oplus, *, 0, 1)$ is said to be *an MV(C)-algebra* if A is an MV-algebra and in addition it satisfies the identity: $2(x^2) = (2x)^2$.

Perfect MV-algebras are those $MV(C)$–algebras generated by their infinitesimal elements or, equivalently, generated by their radical, where radical is the intersection of all maximal ideals. The variety generated by all perfect MV–algebras is also generated by a single MV-chain, actually the MV–algebra C, defined by Chang. The algebra C, with generator $c \in C$, is isomorphic to $\Gamma(Z \times Z,(1, 0))$, with generator (0, 1).

Theorem 1. *An* 1*-generated free MV* (C)*-algebra* $F_{MV(C)}(1)$ *is isomorphic to* C^2 *with free generator* (c, c^*).

Theorem 2. *Any m-generated finitely presented MV* (C)*-algebra* A *is projective.*

Theorem 3. *Any m-generated chain* $MV(C)$*-algebra* C_k $(k \leq m)$ *is projective.*

Theorem 4. *Equational class of MV*(C)*-algebras has unitary unification type.*