Optimization of an initial element for the linear neutral functional differential equation with the mixed initial condition

Tamaz Tadumadze

e-mail: tamaz.tadumadze@tsu.ge
Department of Mathematics, Iv. Javakhishvili Tbilisi State University, 11 University St., 0186 Tbilisi, Georgia

For the system of neutral functional differential equations

$$\dot{x}(t)=\begin{pmatrix}\dot{y}(t)\\ \dot{z}(t)\end{pmatrix}=A(t)x(t)+B(t)y(t-\tau)+C(t)z(t-\sigma)+D(t)\dot{x}(t-\eta),t\in[t_0,t_1],$$

with the mixed initial condition

$$\begin{cases}y(t)=\varphi(t),t\in[t_0-\tau,t_0),y(t_0)=y_0,\\ z(t)=g(t),t\in[t_0-\sigma,t_0],\\ \dot{x}(t)=h(t),t\in[t_0-\eta,t_0),\end{cases}$$

an optimization problem of initial element

$$(t_0,\tau,\sigma,y_0,\varphi(\cdot),g(\cdot),h(\cdot))\in[t_{01},t_{02}]\times[\tau_1,\tau_2]\times[\sigma_1,\sigma_2]\times Y\times\Phi\times G\times H$$

is investigated in the case of nonlinear boundary conditions and functional. Necessary conditions of optimality are proved by a method given in [1]: for initial moment and delays in the form of equalities and inequalities, for initial functions in the form of integral maximum principle. Obtained results are concretized for functionals which are using in the inverse problems. Analogous problem is studied in [2] for the case of the discontinuous initial condition.

References

[1] G. L. Kharatishvili, T. A. Tadumadze, Variation formulas of solutions and optimal control problems for differential equations with retarded argument. *J. Math. Sci. (NY)* , **104**, 1(2007), 1-175 .

[2] T. Tadumadze, Optimization of initial data for linear neutral functional differential equations with the discontinuous initial condition. *Azerbaijan Journal of Mathematics*, **2**, 2 (2012), 84-93.